

DISCRETE MODEL OF IDEOLOGICAL STRUGGLE ACCOUNTING FOR MIGRATION

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Abstract

A discrete in time model of ideological competition is formulated taking into account population migration. The model is based on interactions between *global* populations of non-believers and followers of different ideologies. The complex dynamics of the attracting manifolds is investigated. Conversion from one ideology to another by means of (i) mass media influence and (ii) interpersonal relations is considered. Moreover a different birth rate is assumed for different ideologies, the rate being assumed to be positive for the reference population, made of initially non-believers. Ideological competition can happen in one or several regions in space. In the latter case, migration of non-believers and adepts is allowed; this leads to an enrichment of the ideological dynamics. Finally, the current ideological situation in the Arab countries and China is commented upon from the point of view of the presently developed mathematical model. The massive forced conversion by Ottoman Turks in the Balkans is briefly discussed.

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1 Nonlinear systems, population dynamics, and opinion formation

Competition among ideologies for the people minds is an interesting phenomenon that can lead to large consequences for the social structure and for the social evolution of populations. Religious wars in the past or social revolutions during the XX-th century are closely connected to such ideological competitions. Ideological competition is possible because people might and can accept or drop ideas and systems of ideas and thereafter act according to the newly adopted system of ideas [1, 2]. Population of believers or non-believers do form. The number of individuals in these populations can change with time. Thus the ideological competition is closely connected to population dynamics state but the human ideological struggle has its specific features: animals cannot change their kind (zebras cannot become lions; electrons cannot become bosons) but people can (though sometimes may not) change their ideology.

The systems studied as population dynamics are among the best examples of nonlinear systems in quantitative social and natural sciences [3]. Different kinds of nonlinearities arise in the model equations of such systems, for example (i) because of interactions among the individuals or the populations or (ii) because of limitations in the environment [4, 5, 6, 7, 8, 9]. Such nonlinear model systems require applying methods of nonlinear dynamics [10, 11, 12, 13, 14], chaos theory [15, 16, 17, 18], theory of stochastic processes [19, 20, 21] and even variational and other methods of the theory of turbulence [22, 23, 24].

In this paper, ideological struggle will be considered to occur on a two-dimensional surface (regions in a country or regions containing an arbitrary number of countries). We shall assume that the population of the corresponding territory is divided into sub-populations of followers of several ideologies and one population of non-believers. Any newborn, whatever each parent ideology is necessarily at first a non-believer; any ideology is acquired later. This assumption has as a consequence that the rate of change of the number of individuals by birth and death can be positive only for the population of non-believers. The corresponding rate for the sub-populations of the followers of the different ideologies is necessarily negative as, for these, only death is possible.

What will be apparently new in our model is the possibility of migration of groups of non-believers or followers of ideologies. Indeed it is going to be assumed that non-believers or followers of different ideologies can move from one region to another one, even if the regions do not have a common boundary: thus allowing motion by airplanes or due to a transit crossing of a region by means of train, ship, bus, car, bicycle, horse, camel, donkey, etc.. This movement can change its intensity in time; moreover the birth rates and conversion coefficients are allowed to change as well. The result of this is a very complicated dynamics in time and space.

The model system of nonlinear equations is taken as a system of difference equations which means that one assumes discrete time. One fundamental reason for this is our wish that the model be close to the true situation as much as

possible, in the sense that data is usually collected for discrete time intervals and for a finite number of regions. These two features of the real social life are thus accounted for in the model below.

The paper is organised as follows. In Sect. 2, the system of model equations is formulated for the case of a single region, for simplicity and for outlining the notations. The possibility of time change of the coefficients of the model equation leads us to the notion of *special points*. Special points of the system of these model equations are calculated on the basis of the rules used for calculating so called fixed points. However as the coefficients of the system vary in time, the points are not truly fixed but move in the phase space. Because of this, we use for such points the name special points. If the coefficients of the model system of equations do not change in time, the special points become usual fixed points.

Sect. 3 is devoted to a discussion of the model for the case of two, obviously adjoining, regions. The possibility of migration being allowed, the dynamics become more complicated. Two particular cases are discussed: massive leaving of non-believers from one of the regions to the other, in Sect. 3.1 and massive invasion by followers of some ideology from one of the two regions to the other in Sect. 3.2. Sect. 4 is devoted to the formulation of the model for an arbitrary number of regions and an arbitrary number of ideologies. On the basis of the general model we discuss the possibilities for the strengthening or weakening of an official ideology. Sect. 5 contains several pertinent remarks. Examples are taken from recent and non-recent history : Arab countries and China, and the spreading of Islam in Europe from the beginning of the 15th century though the reader can be aware of other cases according to its own knowledge of history. Other discussions and conclusions are found based on a summary in Sect. 6.

2 Single region

2.1 Model

In order not to overload the basic equations with many terms, let us first present the model and some discussion in the case of a single region. In this region, consider that there are $n+1$ population types, or ideologies: one specific population of non-believers, or having *no* opinion at all about some subject, with spatial density $\rho_0^{(t)}$ of members at time t , and n populations with spatial densities $\rho_i^{(t)}$ of followers of different ideologies ($i = 1, \dots, n$) about some given question (the measure of all densities is people per square kilometer). There is no restriction to considering only *political ideologies*; a religion can be taken as an ideology as well.¹ Notice that i can be considered as an index indicating the component of the *degree of freedom vector* characterizing an individual. However, in view of being more general, one can consider that some i corresponds to a population,

¹Languages are not here considered to be ideologies, due to our consideration that one can only belong to one ideology at a given time, thus we exclude multilingual populations for our present considerations.

having some mere (so called scientific) degree of freedom for characterising part of the population, i.e. e.g. characterised by by a political *and* a philosophical attitude, like distinguishing between ($i = 1$) socialist Buddhists and ($i = 2$) liberal Buddhists, or ($i = 3$) socialists jews and ($i = 4$) liberal jews, etc.² Thus the total density at time t is

$$\rho^{(t)} = \rho_0^{(t)} + \sum_{i=1}^n \rho_i^{(t)} \quad (1)$$

The model (difference) equations for describing the interactions among the above $n + 1$ populations are assumed to be of the Lotka-Volterra type

$$\rho_i^{(t+1)} - \rho_i^{(t)} = r_i^{(t)} \rho_i^{(t)} + \sum_{j=0}^n f_{ij}^{(t)} \rho_j^{(t)} + \sum_{j=0}^n b_{ij}^{(t)} \rho_i^{(t)} \rho_j^{(t)} \quad (2)$$

where $r_i^{(t)}$ is the "biological" rate for changing the density of the corresponding population by births and deaths. In other words $r_i^{(t)}$ accounts for the finite duration of the life of the people. In the present work, no time delay, memory or anticipation is considered between various causes and effects. The change in population number occurs at each unit of time t which is discrete. As already mentioned in the previous section below we assume that only the population of non-believers can have a positive $r_0^{(t)}$ rate: thus we assume that (i) all newborn babies are non-believers and (ii) for this population, the number of births can be larger than the number of deaths. For the other populations of believers, the rates $r_i^{(t)}$, ($n \geq i \geq 1$) are taken to be negative, since only death is possible there.

In addition to births and deaths, we assume in the second and third term of the equation r.h.s. that *two* kinds of conversion exist in the system of populations. The first kind of conversion is triggered by the mass media: newspapers, Internet, TV, radio, etc. In the model equation for the i -th population, the mass media conversion is accounted for by the term $f_{ij}^{(t)} \rho_j^{(t)}$ where $f_{ij}^{(t)}$ is the corresponding coefficient of conversion influencing the number of followers of the j -th ideology who are converted to the i -th ideology because of mass media influence.

The second kind of conversion is the contact conversion that happens, e.g., after conversations between members of different ideologies. We assume for simplicity that the result from this kind of conversion is proportional to the densities of the conflictual contacting ideologies. Thus this kind of conversion is modelled by the term $b_{ij}^{(t)} \rho_i^{(t)} \rho_j^{(t)}$, where $b_{ij}^{(t)}$ is a coefficient that accounts for the intensity of the conversion.

The dynamics of the system of $n + 1$ populations as described by Eq. (2) can be discussed in an $n + 1$ -dimensional Cartesian phase space. The coordinates on the axes spanning this space correspond to the densities $\rho_0, \rho_1, \dots, \rho_n$ of the

² Of course other notations can be used, like $\rho_{i,j,k,l,\dots}$ where i indicates the ideology, j the religion, k the sex, l the social status, \dots , etc.

adepts of the corresponding ideology. The coordinates $\rho_0, \rho_1, \dots, \rho_n$ define a point in this phase space denoting the social state of the investigated system at a given time. As the time t evolves, the system state follows a trajectory: the so called *system trajectory*. This system trajectory can be attracted to different kinds of manifolds in the phase space. These manifolds are called attractors: they can be points, or more complicated structures. Most of the time the system is on its way to some attracting manifold, which in most cases can be some equilibrium state.

2.2 Spreading of ideologies

Let us now discuss the simplest case, i.e. spreading of one ideology in one area due to opinion competition. For simplifying the presentation, let only two populations/opinions/ideologies existing: (i) the non-believers ρ_0 and (ii) the followers ρ_1 of ideology 1. The model equations read

$$\rho_0^{(t+1)} - \rho_0^{(t)} = r_0^{(t)} \rho_0^{(t)} + \sum_{j=0}^1 f_{0j}^{(t)} \rho_j^{(t)} + \sum_{j=0}^1 b_{0j}^{(t)} \rho_0^{(t)} \rho_j^{(t)} \quad (3)$$

$$\rho_1^{(t+1)} - \rho_1^{(t)} = r_1^{(t)} \rho_1^{(t)} + \sum_{j=0}^1 f_{1j}^{(t)} \rho_j^{(t)} + \sum_{j=0}^1 b_{1j}^{(t)} \rho_1^{(t)} \rho_j^{(t)} \quad (4)$$

Without any loss of generality, let also $f_{00} = f_{11} = 0$ and $b_{00} = b_{11} = 0$, i.e. the mass media influence and binary conversion do not increase the number of members of both populations *per se* through some miraculous multiplication/addition. Then the system of equations reduces to

$$\rho_0^{(t+1)} - \rho_0^{(t)} = r_0^{(t)} \rho_0^{(t)} + f_{01}^{(t)} \rho_1^{(t)} + b_{01}^{(t)} \rho_0^{(t)} \rho_1^{(t)} \quad (5)$$

$$\rho_1^{(t+1)} - \rho_1^{(t)} = r_1^{(t)} \rho_1^{(t)} + f_{10}^{(t)} \rho_0^{(t)} + b_{10}^{(t)} \rho_1^{(t)} \rho_0^{(t)}. \quad (6)$$

Remember that $r_1^{(t)}$ is imposed to be negative and $r_0^{(t)}$ positive.

The special phase space points connected to the dynamics of the system, Eqs. (5) and (6), are obtained when the left-hand sides of the two equations (5), (6) are equal to 0. We shall call such points special points instead of fixed points because of the following reason. Fixed points do not move in the phase space, but the special points can move in this space, because the system parameters can change with time. Only the special point corresponding to the extinction of all populations is a fixed point for all values of the system parameters.

The (two) special points for the system of equations (5) and (6) are

$$\rho_0^{(t)} = \rho_1^{(t)} = 0 \quad (7)$$

and the non-trivial special point

$$\rho_0^{(t)} = \frac{f_{10}^{(t)} f_{01}^{(t)} - r_1^{(t)} r_0^{(t)}}{r_0^{(t)} b_{10}^{(t)} - b_{01}^{(t)} f_{10}^{(t)}}, \quad \rho_1^{(t)} = \frac{f_{10}^{(t)} f_{01}^{(t)} - r_1^{(t)} r_0^{(t)}}{r_1^{(t)} b_{01}^{(t)} - b_{10}^{(t)} f_{01}^{(t)}}. \quad (8)$$

N.B. If $r_0^{(t)} b_{10}^{(t)} - b_{01}^{(t)} f_{10}^{(t)} = 0$ or $r_1^{(t)} b_{01}^{(t)} - b_{10}^{(t)} f_{01}^{(t)} = 0$, there exists a single special point $\rho_0^{(t)} = \rho_1^{(t)} = 0$.

The first special point $\rho_0^{(t)} = \rho_1^{(t)} = 0$ corresponds to the extinction of the (two) populations. We note that this special point, when reached after some trajectory depending on the initial conditions, does not move in phase space: it is a fixed point. The second special point is much more interesting. In general, the coefficients in the model equations (5), (6) can change their values in the course of time. Thus, the second special point can move in the phase space. As the phase point corresponding to the system state moves, one can observe an interesting *hunt* for equilibrium: the phase point 'hunts' the special point. The dynamics of the system trajectory in the phase space can become quite complicated, as seen in Fig. 1, for a particular numerical example. Changing r_1 leads to additional movement of the special point as shown in Fig. 2.

As we can see the system trajectory (marked with the dashed line in Fig. 1) has to make a (long) journey through the phase space before finding and reaching the non-zero state described by the moving special point. This occurs despite the fact that at the initial time the system state and the special point are not very far one from another. Thus, the possibility that a time change in system coefficients enriches the dynamics in comparison to the case of constant coefficients of the model equations is easily demonstrated.

Quite interesting are the phenomena connected to the linear stability of the two special points from Eqs.(7)-(8). The eigenvalues connected to the stability of the special point (0, 0), which is a fixed point, are

$$\lambda_{1,2}^{(t)} = \frac{1}{2} \left[r_0^{(t)} + r_1^{(t)} \pm \sqrt{(r_0^{(t)} - r_1^{(t)})^2 + 4f_{01}^{(t)} f_{10}^{(t)}} \right]. \quad (9)$$

Let us assume that the coefficients connected to the mass media conversion are both positive, i.e. $f_{01}^{(t)} > 0$ and $f_{10}^{(t)} > 0$. Then the two eigenvalues are real. If the system parameters are constant in time and $|\lambda_1| < 1$ and $|\lambda_2| < 1$ the special point is stable. But as the parameters can vary with time the condition $|\lambda_1| < 1$ and $|\lambda_2| < 1$ can be violated and the point may become unstable for some time interval. If it is unstable, it cannot be reached.

In addition a special point may move periodically; whence any attempt for the system trajectory to catch the moving special point can lead to a complex motion of the phase trajectory in the phase plane. This can be illustrated on the basis of the following example. Let $f_{01} = 0$ and $b_{10} = 0$ and fix the values of parameters as mentioned in the caption of Figs. 3-4. Let all parameters be constant, except for r_0 which oscillates. The result is an oscillating special point; the system trajectory moves in a complex way in the phase space (see Fig. 3).

Of course, there can be more than one ideology, and the so called non-believers, present in the region. Then the existing ideologies can compete for adepts, such that the dynamics of the state point in the phase space can become more complicated. An illustration of such a case is shown in Fig. 5 for a particular numerical example.

3 The case of two regions

Consider two regions, I and II, and some possibility for migration of believers or non-believers between the regions. Let us note the $n + 1$ densities of populations/opinions in the first region as $\rho_{I,0}, \rho_{I,1}, \dots, \rho_{I,n}$ and the densities of populations in the second region as $\rho_{II,0}, \rho_{II,1}, \dots, \rho_{II,n}$ (some of $\rho_{I,i}$ and $\rho_{II,j}$ can be 0). Let it be conventionally accepted from now on that the label before the comma in the index refers to the region, while the index after the comma refers to the ideology type or population. The leaving from and coming to a region are accounted for by introducing new terms in the model equations with appropriately defined specific diffusion coefficients. Let the coefficient of leaving (L) the area I toward the area II by adepts of the i -th population be $L_{I \rightarrow II,i}$ and the coefficient of leaving of the area II toward the area I by the adepts of the i -th population be $L_{II \rightarrow I,i}$. The corresponding coefficients for the incoming (I) followers of the i -th population are $I_{II \rightarrow I,i}$ and $I_{I \rightarrow II,i}$, or $I_{I \leftarrow II,i}$ and $I_{II \leftarrow I,i}$.³ A necessary balance exists between the number of adepts leaving one area when going into the other, implying some obvious relationships between these coefficients. We assume that an adept does not change his/her opinion by simply leaving from or going into a region.

The model equations for the two area ($n + 1$) population number evolutions become

$$\begin{aligned} \rho_{I,i}^{(t+1)} - \rho_{I,i}^{(t)} = & r_{I,i}^{(t)} \rho_{I,i}^{(t)} + \sum_{j=0}^n f_{I,ij}^{(t)} \rho_{I,j}^{(t)} + \sum_{j=0}^n b_{I,ij}^{(t)} \rho_{I,i}^{(t)} \rho_{I,j}^{(t)} - \\ & L_{I \rightarrow II,i}^{(t)} \rho_{I,i}^{(t)} + I_{I \leftarrow II,i}^{(t)} \rho_{II,i}^{(t)} \end{aligned} \quad (10)$$

$$\begin{aligned} \rho_{II,i}^{(t+1)} - \rho_{II,i}^{(t)} = & r_{II,i}^{(t)} \rho_{II,i}^{(t)} + \sum_{j=0}^n f_{II,ij}^{(t)} \rho_{II,j}^{(t)} + \sum_{j=0}^n b_{II,ij}^{(t)} \rho_{II,i}^{(t)} \rho_{II,j}^{(t)} - \\ & L_{II \rightarrow I,i}^{(t)} \rho_{II,i}^{(t)} + I_{II \leftarrow I,i}^{(t)} \rho_{I,i}^{(t)} \end{aligned} \quad (11)$$

Let us consider, for illustrating the notations, the particular case of exchange between two areas when only two populations/opinions are present in each region: the non-believers ρ_0 and followers of the (so called prevailing) ideology 1,

³It might be interesting to have one or several indices attached to L and I in order to distinguish among the migration causes like migration currents due to external forces or economic reasons due to gradients in standard of living, though this would make the notations rather elaborate. Such a remark is highly appropriate for the cases discussed below. We thank one of the reviewers for such a good suggestion.

i.e. ρ_1 . The model equations are reduced to

$$\begin{aligned}
\rho_{I,0}^{(t+1)} - \rho_{I,0}^{(t)} &= r_{I,0}^{(t)} \rho_{I,0}^{(t)} + f_{I,01}^{(t)} \rho_{I,1}^{(t)} + b_{I,01}^{(t)} \rho_{I,0}^{(t)} \rho_{I,1}^{(t)} - L_{I \rightarrow II,0}^{(t)} \rho_{I,0}^{(t)} + I_{I \leftarrow II,0}^{(t)} \rho_{II,0}^{(t)} \\
\rho_{I,1}^{(t+1)} - \rho_{I,1}^{(t)} &= r_{I,1}^{(t)} \rho_{I,1}^{(t)} + f_{I,10}^{(t)} \rho_{I,0}^{(t)} + b_{I,10}^{(t)} \rho_{I,1}^{(t)} \rho_{I,0}^{(t)} - L_{I \rightarrow II,1}^{(t)} \rho_{I,1}^{(t)} + I_{I \leftarrow II,1}^{(t)} \rho_{II,1}^{(t)} \\
\rho_{II,0}^{(t+1)} - \rho_{II,0}^{(t)} &= r_{II,0}^{(t)} \rho_{II,0}^{(t)} + f_{II,01}^{(t)} \rho_{II,1}^{(t)} + b_{II,01}^{(t)} \rho_{II,0}^{(t)} \rho_{II,1}^{(t)} - L_{II \rightarrow I,0}^{(t)} \rho_{II,0}^{(t)} + I_{II \leftarrow I,0}^{(t)} \rho_{I,0}^{(t)} \\
\rho_{II,1}^{(t+1)} - \rho_{II,1}^{(t)} &= r_{II,1}^{(t)} \rho_{II,1}^{(t)} + f_{II,10}^{(t)} \rho_{II,0}^{(t)} + b_{II,10}^{(t)} \rho_{II,1}^{(t)} \rho_{II,0}^{(t)} - L_{II \rightarrow I,1}^{(t)} \rho_{II,1}^{(t)} + I_{II \leftarrow I,1}^{(t)} \rho_{I,1}^{(t)}
\end{aligned} \tag{12}$$

From the numerous possible scenarii resulting from the above, we shall further restrict the present considerations to two cases only:

- massive leaving of one region by non-believers ($i = 0$) under the pressure of the followers of the prevailing ideology ($i = 1$); e.g., protestants leaving Europe because of catholics pressure in the XVI -th century or so.
- massive invasion of one region by the followers of the prevailing ideology ($i = 1$) from the other region; e.g., catholics conquistadors going to South-America in the XV-th century or so.

N.B. Although both cases appear at first rather equivalent, the two cases under interest should nevertheless be suggestively markedly different to the astute reader, because of the birth/death rate conditions imposed on populations 0 and 1 respectively , from the mere beginning.

3.1 Mass departure

Let the followers of the prevailing ideology ($i = 1$) put pressure on the non-believers ($i = 0$) in order to fully convert some region toward their ideology; thus let the a massive leaving of one region, e.g. I, by non-believers ($i = 0$) toward the other region, i.e. II. We assume that no exchange of populations between the two regions happens except for the leaving of non-believers from region I to region II, i.e. $L_{I \rightarrow II,0}^{(t)} \neq 0$ and all other coefficients L and I , except of course $I_{I \rightarrow II,0}^{(t)}$, in Eq. (12), equal to 0.

From the first equation of the system (12), the condition for a decrease in the number of non-believers ($i = 0$), in I, is

$$L_{I \rightarrow II,0}^{(t)} > r_{I,0}^{(t)} + f_{I,01}^{(t)} \frac{\rho_{I,1}^{(t)}}{\rho_{I,0}^{(t)}} + b_{I,01}^{(t)} \rho_{I,1}^{(t)}. \tag{13}$$

Eq. (13) tells us that if the coefficient $L_{I \rightarrow II,0}^{(t)}$ describing the leaving is large enough in order to compensate the birth rate and the effect of the conversion mechanisms, then the population of non-believers in the region I will start to decrease. After some time, the followers of the ideology ($i = 1$) can thus become the majority of the population in that region.

Due to the balance condition, i.e. the conservation of the number of non-believers in both regions, the number of non-believers in the region II will increase. Thus, from the third equation in (12), one has similar condition on $L_{I \rightarrow II,0}^{(t)}$, but relevant for the behavior of populations ($i = 0$ and $i = 1$) in region II, i.e.

$$L_{I \rightarrow II,0}^{(t)} \equiv I_{II \leftarrow I,0}^{(t)} > -\frac{r_{II,0}^{(t)}\rho_{II,0}^{(t)} + f_{II,01}^{(t)}\rho_{II,1}^{(t)} + b_{II,01}^{(t)}\rho_{II,0}^{(t)}\rho_{II,1}^{(t)}}{\rho_{I,0}^{(t)}}. \quad (14)$$

3.2 Mass invasion

Another scenario leading to some domination of a region by an ideology may be due to the invasion of that region from another region by the followers of the prevailing ideology in the latter region. Notice that this scenario is more complex than the scenario of leaving in Sec. 3.1 as it requires common efforts by the followers of the ideology in the two considered regions. Let, in Eq. (12), $I_{I \leftarrow II,1}^{(t)} \equiv I_{II \rightarrow I,1}^{(t)} \neq 0$ and all other coefficients L and I in (12) be 0, except of course $L_{II \rightarrow I,1}^{(t)}$. The condition for an increase in the number of believers ($i = 1$), in region I, is

$$I_{I \leftarrow II,1}^{(t)} > -\frac{r_{I,1}^{(t)}\rho_{I,1}^{(t)} + f_{I,10}^{(t)}\rho_{I,0}^{(t)} + b_{I,10}^{(t)}\rho_{I,1}^{(t)}\rho_{I,0}^{(t)}}{\rho_{II,1}^{(t)}}. \quad (15)$$

In other words, the invasion has to compensate for the death rate (remember that $r_{I,1}^{(t)}$ is negative) and is supported by the conversion mechanisms, through $f_{I,10}^{(t)}$ and $b_{I,10}^{(t)}$. Thus, if the followers of an ideology manage to organise mass invasion combined with massive media propaganda (large $f_{I,10}^{(t)}$) and large conversation conversion (large $b_{I,10}^{(t)}$) and if they face a "tired" population of non-believers (small or even negative $r_{I,0}^{(t)}$) which does not take measures to decrease the number of followers of the ideology by conversion (small $f_{I,01}^{(t)}$ and $b_{I,01}^{(t)}$) then the invasion could be successful. The balance condition, i.e. the conservation of the number of ideology ($i = 1$) adepts in both regions, implies that the number of the latter in region I will increase. Thus, from the fourth equation in (12), one has similar condition on $L_{II \rightarrow I,1}^{(t)}$, but relevant for the behavior of populations ($i = 0$ and $i = 1$) in region II, i.e.

$$L_{II \rightarrow I,1}^{(t)} > r_{II,1}^{(t)} + f_{II,10}^{(t)} \frac{\rho_{II,0}^{(t)}}{\rho_{II,1}^{(t)}} + b_{II,10}^{(t)} \rho_{II,0}^{(t)}. \quad (16)$$

4 Arbitrary number of regions. Further comments

The final complication of the model with respect to the number of discussed regions is to consider arbitrary number of regions. We separate the entire studied two-dimensional area into S_k regions, $k = \text{I, II}, \dots, K$. In each region we assume $i = 0, 1, \dots, n$ populations with densities $\rho_{k,i}$ where $i = 0$ denotes the population of non-believers in any ideology. In general we allow migration of members of populations between any two of the areas even between those that do not have common boundary. The corresponding exchange coefficients are denoted as $L_{k \rightarrow l, i}$ and $I_{k \leftarrow l, i}$. The system of $K \cdot (n + 1)$ model equations reads⁴

$$\begin{aligned} \rho_{k,i}^{(t+1)} - \rho_{k,i}^{(t)} = & r_{k,i}^{(t)} \rho_{k,i}^{(t)} + \sum_{j=0}^n f_{k,ij}^{(t)} \rho_{k,j}^{(t)} + \sum_{j=0}^n b_{k,ij}^{(t)} \rho_{k,i}^{(t)} \rho_{k,j}^{(t)} - \\ & \sum_{l=1}^K L_{k \rightarrow l, i}^{(t)} \rho_{l,i}^{(t)} + \sum_{l=1}^K I_{k \leftarrow l, i}^{(t)} \rho_{l,i}^{(t)} \\ & i = 0, 1, \dots, n; \quad k = \text{I, II}, \dots, K \end{aligned} \quad (17)$$

This system describes the dynamics in a $K \cdot (n + 1)$ - dimensional phase space where numerous special points and complicated attracting manifolds arise, vanish, and move with time. The system trajectory can be very complicated as it travels in a high-dimensional phase space in order to come close to some attracting manifold.

5 Practical Remarks

At this stage, many interesting research questions arise about applications: for example, questions on the control of the dynamics of the ideological interaction. Let us discuss a situation on the control of the spreading of ideologies in a selected region of a territory consisting of several sub-regions on the basis of the general model system Eqs.(17). For bettering such a control, the dimension of the phase space of the system should be reduced. This can happen by regulating and reducing the inflow of followers of unwanted ideologies, i.e. technically, by lowering the values of $I_{l \leftarrow k, i}^{(t)}$. An outflow of followers of unwanted ideologies can be stimulated, if this does not affect the coefficients of mass media conversions. Otherwise the outflow has to be reduced also, i.e. lowering $L_{k \rightarrow l, i}^{(t)}$. Without significant inflow and outflow in the selected region, the dimension of the phase space of the ideological dynamics is practically reduced. Within a purpose of control, one has to deal with the followers of the ideologies and with the non-believers in the region.

⁴It is surely obvious to the reader that a few coefficients are necessarily equal to 0, but all terms are hereby written in order to maintain the number of notations at a reasonable level.

The dynamics of the corresponding populations is determined by the birth and death rate of the non-believers as well as by the death rates of the followers of all the other ideologies and by the conversion coefficients. Anyone's experience shows that the conversion by conversation is much part of an e.g. religion conversion in a dynamical context [25, 26, 27, 28, 29, 30], while the conversion due to media impact is well known to be the primary cause of evolution in politics [31, 32, 33, 34]. Of interest is also the conversion of an opinion about goods and consumer products [35, 36, 37, 38]. For completeness, recall that external (propaganda) influence and socio-economic pressure can be mapped into *social forces* [39]. The correspondence with our approach is left for another publication. Nevertheless, they would be easily apparent if one was developing the rhs of the complete dynamical equations

$$\rho_{k,i}^{(t+2)} - 2\rho_{k,i}^{(t+1)} + \rho_{k,i}^{(t)} = \dots, \\ i = 0, 1, \dots, n; \quad k = \text{I, II}, \dots, K \quad (18)$$

Thereafter, one can discuss qualitatively cases of recent ideological control, for example in Arab countries and in China. Massive invasion will be considered at an earlier time, i.e. the XIV-th century invasion of Europe by Turks.

Recently, the failure of ideological control has been seen in several Arab countries. Several reasons for this come in mind on the basis of the discussed model. Let in this case the index i in the model equations denotes: $i = 0$ - no ideology; $i = 1$ - official ideology; $i = 2$ - Islamism; $i = 3$ - liberal ideology; $i = 4$ - moderate Islam. The official ideology connected to corresponding dictatorship regimes has gained followers mostly through the domestic mass media propaganda and conversations supported by financial support.⁵ This led to significant positive value of f_{10} . b_{10} had significant positive value too. Islamists were not successfully converted to the official ideology, i.e., the values of f_{12} and b_{12} were small and negative. More successful was the conversion from the liberal to official ideology which means that f_{13} and b_{13} had positive values. Relatively successful was the conversion from Islam to the official ideology (as the official ideology was partially based on Islam). As consequence of this the coefficients f_{14} and b_{14} had significant positive values. Thus the strategy of the corresponding government was to convert as many as possible non-believers and followers of the Islam to the official ideology and to decrease as much as possible the conversion of the followers of the official ideology to Islamism. As well as the government had enough money and propaganda power it was possible to follow such a strategy.

The liberal ideology spread by the mass media was mainly connected to the west technology: Interned and West TV channels and military shows. This

⁵There is also the "service and welfare support" to the need of the population, - which is not really mass media propaganda nor conversation, but rather an opinion "formation" due to some "theoretically gratuitous altruism" [40, 41, 42, 43, 44] by "charitable organisations" following resources transfer, with political goals [45, 46, 47, 48, 49]. In view of its clustering effect, somewhat a group response to group action, it might be considered to be a higher order term in Eqs.(2), or still be integrated into each $f_{ij}^{(t)}$ coefficient.

ideology was not able to compete successfully against the government ideology which means that the corresponding conversion coefficients f_{30}, b_{30} had smaller values in comparison to f_{10}, b_{10} . The conversion from official ideology to liberal ideology was not successful, i.e., f_{31} and b_{31} had negative values. The values of f_{32} and b_{32} were very small negative (there was no conversion of Islamists to liberal ideology and small number of followers of liberal ideology converted to Islamism). The conversion of the followers of the Islam to liberal ideology was also not successful. Thus the values of f_{34} and b_{34} were almost zero. Thus the strategy of the followers of the liberal ideology was to convert the non-believers (mainly young people) and to minimize the decreasing of the followers by conversion to the official ideology and Islamism.

The Islamism gained followers by conversations, hypocritical altruism and the domestic mass media. Using the power of the Islam large values of f_{20}, f_{24} and b_{20}, b_{24} have been maintained. f_{21}, b_{21} had small positive values as some supporters of the official ideology converted to Islamism. The values of f_{23} and b_{23} were also positive but much smaller as very few supporters of the liberal ideology converted to Islamism.

The followers of the moderate Islam increased in the course of years because of the large conversion rates f_{40}, b_{40} from (youth) non-believers to followers of the Islam. Much smaller conversion happened from Islam to followers of the official ideology (small negative values of f_{41} and b_{41}) and even smaller was the conversion from the followers of the liberal ideology (very small positive values of f_{43} and b_{43}). The conversion of fanatic Islamists to the moderate Islam was not successful (negative values of f_{42} and b_{42}). The centuries old strategy of the followers of moderate Islam to convert very successfully (young) non-believers worked very well and compensated the losses due to conversion to official ideology and Islamism.

The outflow of people from these countries toward others was not (at this time) large enough, though the large birth rates in the Arab countries produced a large amount of jobless youth which becomes resistive to accept the official ideology in the corresponding Arab country. Conversion to the corresponding official ideology by conversation, was apparently not very successful either and the values of f_{10} and b_{10} remained positive but decreased very much with respect to the values of the corresponding coefficients for the other ideologies. Thus in the course of the years the non-believers were converted much more successfully to moderate Islam, Islamism and even to the liberal ideology. The spreading of liberal opinions following mass media conversion, such as Internet, Facebook, Twitter, etc., was not totally controlled and thus the non-believers were even more resistive for conversion to the official ideology. Thus the official (and mostly totalitarian) ideology in these countries had decreased percentage of followers and finally collapsed. Now in these countries we observe fast growth of the fanatic Islamists, many moderate followers of Islam, still influential population of followers of the former official ideology and a slowly increasing amount of people that "convert" to some liberal ideology, under the influence of the mass media from the West. Then the future development in these societies in the next years will depend much on the competition between the liberal ideol-

ogy, Islamists and moderate Islam followers. The liberal ideology has a problem: it can not convert very successfully the followers of the moderate Islam. The Islamists do not have this problem. Thus a possible capsulation of the Arab societies (and especially the decreasing influence of the West mass media) could lead to a dominance of Islamists and Islamic ideology which is quite deeply rooted in these societies and has a large tradition of conversion of non-believers.

Much more successful is the ideological control in China. Many elements of the capitalist economic system help for successfully absorbing the inflow of young people who have studied in the West. This absorption is easily transformed by (social or economic or political) pressure conversion in the official Chinese ideological system. Birth rates are much under control and the outflows of followers of different ideologies are much regulated [50].

Let in the case of China the value of the index i denote ideologies as follows: $i = 0$ - no ideology; $i = 1$ - official ideology; $i = 2$ - liberal ideology. Mass media is under control and the Chinese government takes steps toward controlling the Internet and the connections between social networks. This ensures large positive values of the conversion coefficients f_{10} and b_{10} whereas the values of the corresponding coefficients for the liberal ideology f_{20} and b_{20} are positive but much smaller. As the official ideology contains element of liberalism the values of conversion coefficients f_{12} and b_{12} are small positive and the values of corresponding coefficients for the liberal ideology f_{21} and b_{21} are negative. We note that in principle the conversation conversion is much out of control⁶ but up to now it is successfully countered by governmental propaganda and internal social pressure and the value of b_{20} is small. Thus the ideological situation in China is stable and the ideological dynamics is dominated by the official ideology. As no large demographic problem is expected in the next decades and if the economic development continues, the stability of the ideological situation in China is expected to remain largely unchanged.

Finally, from the historic situations connection to mass invasion of followers of some ideology to a region dominated by another ideology let us consider the invasion of the Ottoman Turks into Europe that started from the second half of the 14-th century. We consider two regions: region I.: the Balkans (the invaded region); and region II. (Anatolia, Asia Minor and surrounding areas of Asia). At this time the invaded region: the Balkans were populated by followers of Christianity (mostly from the orthodox branch but with presence of followers of catholicism too). Weakened by the 'Black death' plague the small Balkan countries were conquered one after another by the Muslim Ottoman turks who started to impose Islam, ruining churches or transforming these into mosques. Let us discuss qualitatively the situation from the point of view of the model presented above. Let the values of the index i mean the following: $i = 0$ - non-believers; $i = 1$ - Christians; $i = 2$ - followers of the Islam. The birth rates supported the population of non-believers that in course of years converted to Christianity or Islam. At that time there was no mass media as

⁶Some investigation in such a respect is worth to be pointed out: in the Moss model [51], social embededness takes the form of observation of neighbours public consumption activities such as garden watering and car washing as well as word of mouth communication.

we know it today so $f_{I,01}$, $f_{II,01}$, $f_{I,10}$, $f_{II,10}$ were practically 0 and there was no possibility of media influence on non-believers in order to convert them to Christianity. The values of the coefficients $f_{I,02}$, $f_{II,02}$, $f_{I,12}$, $f_{II,12}$ were practically 0 too as there was no conversion of the followers of Islam to Christianity or to the population of non-believers. The public spreading of the administrative orders of the Ottoman administration and the lower taxes for Muslims can be treated as sort of media that contributed to conversion and led to small positive values of the coefficients $f_{I,20}$, $f_{II,20}$, $f_{I,21}$ and $f_{II,21}$. The main process for ensuring the existence of the Christian population was the conversion of (young) non-believers to Christianity by means of conversation, i.e., the value of $b_{I,10}$ was large positive. The value of $b_{II,10}$ was small positive as in the region II the Christian population was surrounded by large Muslim population that put pressure acting in the direction of muslimisation of the non-believers in that region. This was a branch of the Ottoman assimilation mechanism that included displacement from Europe to Asia and then muslimisation. In the two regions there was almost no conversation conversion from Christianity or Islam to the population of non-believers. Because of this the values of the coefficients $b_{I,01}$, $b_{I,02}$, $b_{II,01}$, $b_{II,02}$ were negligibly small. The conversation conversion from Islam to Christianity was also negligibly small, i.e., the values of the coefficients $b_{I,12}$, $b_{II,12}$ were practically 0.

The politically dominating Muslim population was not able to keep large enough level of conversation conversion in the region I: the values of $b_{I,20}$ and $b_{I,21}$ were positive but far away from the needed values for fast conversion of the Christian population to Islam. The population of entire regions refused to convert to Islam. For increasing the value of $f_{I,20}$ the '*enichars*' tactic was used for almost three centuries - Christian children were taken away from their parents and then converted to Islam. The increase of the values of $f_{I,21}$ and $b_{I,21}$ was attempted by campaigns to muslimise entire regions of the Balkan peninsula (today's Albania where much more than the half of population were catholics, today's Bosnia and Herzegovina, and today's Rodopa mountain region in Bulgaria where the population consisted of almost 100 % followers of Orthodox Christianity). Who refused to convert was killed. In the region II (Asia) the situation was different as there the Muslim population was large majority and the pressure for conversion of Christian population was effective, i.e., the values of $b_{II,20}$ and $b_{II,21}$ were positive and large enough for the conversion to happen.

Important point of the Ottoman strategy for muslimisation of the Balkans was connected to migration. There was massive migration of Muslims from region II (Asia) to region I (Balkans) especially after Timur (known also as Tamerlane) defeated and captured the Ottoman sultan Bayezit I in 1402 near today's Ankara. The remains of the Ottoman army were transported from Anatolia (region II) to Thrace (region I) by Venetian ships.⁷ Further migration

⁷Venetians preferred to have the weak (at that time) Ottoman turks as enemy and a buffer against the mighty Timur but Timur aimed to restore the Mongolian Empire of Chengis Khan and turned to conquer China. The Ottoman army could reorganize and crushed the Bulgarian uprising in 1405. Thus the way to Europe was opened for the Ottoman that managed to

followed and the coefficients $L_{II \rightarrow I,2}$ and $I_{I \leftarrow II,2}$ had large positive values. The migration of Muslims in opposite direction was very small and the values of the coefficients $L_{I \rightarrow II,2}$ and $I_{II \leftarrow I,2}$ were practically 0.

Parallel to the migration of the Muslim population the Ottoman turks displaced the Christian population and non-believers (children) and part of it was sent from region I to region II (example of this is the displacing of large part of Christian population of Belgrade to Istanbul) so the coefficients $L_{I \rightarrow II,1}$, $L_{I \rightarrow II,0}$, $I_{II \leftarrow I,0}$, $I_{II \leftarrow I,1}$ had also large positive value. Such population usually remained in Anatolia and had been converted to Islam through the centuries of Ottoman rule. Thus the values of the coefficients $L_{I \rightarrow I,1}$, $L_{II \rightarrow I,0}$, $I_{I \leftarrow II,0}$ and $I_{I \leftarrow II,1}$ were very small.

The conversion strategy of the Ottoman turks acted for a very long time. Because of this, 450 years after the start of the invasion between 30% and 50% of the population of several regions of the Balkans has been practically converted to Islam.

6 Conclusions

In this paper we have discussed a discrete model of ideological interaction which is easy for practical implementation for prognosis of the dynamical evolution of ideological situations. The model system is based on a discrete time process within a Lotka-Volterra framework, but allowing for migration of populations between areas. The model contains specific terms mimicking as much as possible stylised facts or, as usually, common sense in opinion evolution dynamics. Each country or region of countries can be separated into a number of regions for which some data on the number of followers of different ideologies can be collected. Birth and death rates are often available from statistical sources indeed. From these sources estimations of the coefficients of mass leaving and incoming can be usually obtained as well. What remains, and is the most difficult task, is the determination of the conversion coefficients. However it seems that they can be obtained by standard or appropriate statistical procedures, like opinion pool evaluations, or by indirect means of behaviour observation. Of course long time interval observations are required in order to determine the time change of the coefficients for model equations.

After determining the coefficients one can launch the model, starting from Eqs.(17), with known or appropriate available initial conditions. Under the assumption that the coefficients of the model equations vary rather slowly, it can be fairly assumed that either they are constant during several time steps or they change following a linear law during these steps. Therefore, the model is expected to give a valuable approximate picture of the ideological situation in the observed region for these time steps. Of course, such approximations can be best applied when the situation in the corresponding regions is stable and when no large social change happens. In times of large social changes, and extreme

besiege even Vienna two times in the next 300 years.

events, like wars, revolutions, ..., the constant or linear approximation can be unsatisfactory.

As discussed here above, the model admits a very large number of possible scenarios. In conclusion, even without solving in detail the system of equations, one can obtain some orientation about ideology situations in different regions merely on the basis of qualitative discussions of the equations of the model. Only limited to opinions, it can be fruitfully compared with attempts of the so called World Model [52] for socio-economic considerations.

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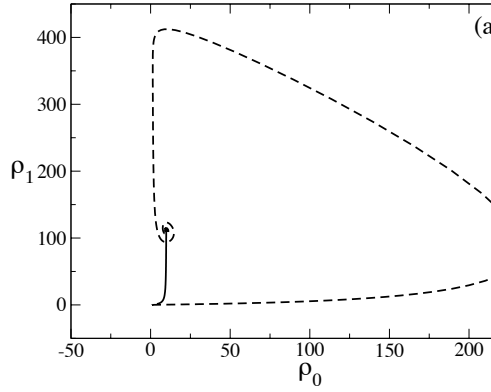


Figure 1: Illustration of the system trajectory and the special points in the phase space for the case of a single area and two populations, - non-believers and followers of an ideology. Solid line: movement of the non-zero special point given by Eq.(8). Dashed line: system trajectory calculated on the basis of Eq.(5) and Eq.(6). The parameters are appropriately chosen in order to illustrate how the system phase point 'hunts' and 'catches' the (non trivial) special point $(\rho_0^{(t)}, \rho_1^{(t)})$ as $t \rightarrow \infty$. Initial values: $\rho_0 = 0.98$, $\rho_1 = 0.02$. The constant parameters are: $f_{10}^{(t)} = 0.0002$; $f_{01}^{(t)} = 0.0001$; $r_1^{(t)} = -0.001$; $b_{01}^{(t)} = -0.0001$; $b_{10}^{(t)} = 0.0001$. All such parameters are assumed to be constant. The only time changing parameter is: $r_0^{(t)} = 0.01 \tanh(0.01t)$. The maximum value of t used is $t = 40000$.

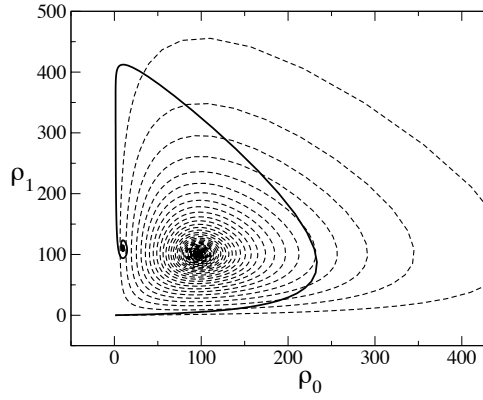


Figure 2: Effect of the change of r_1 . Solid line: the corresponding trajectory from Fig.1. Dashed line: the system trajectory for $r_1^{(t)} = -0.01[1 - 0.01 \tanh(0.001t)]$. Illustration of the system trajectory and the special points in the phase space for the case of a single area and two populations, - non-believers and followers of an ideology. Solid line: movement of the non-zero special point given by Eq.(8). Dashed line: system trajectory calculated on the basis of Eq.(5) and Eq.(6). The parameters are appropriately chosen in order to illustrate how the system phase point 'hunts' and 'catches' the (non trivial) special point $(\rho_0^{(t)}, \rho_1^{(t)})$ as $t \rightarrow \infty$. Initial values: $\rho_0 = 0.98$, $\rho_1 = 0.02$. The constant parameters are: $f_{10}^{(t)} = 0.0002$; $f_{01}^{(t)} = 0.0001$; $b_{01}^{(t)} = -0.0001$; $b_{10}^{(t)} = 0.0001$. All such parameters are assumed to be constant. The only time changing parameter is: $r_0^{(t)} = 0.01 \tanh(0.01t)$. The maximum value of t is $t = 40000$.

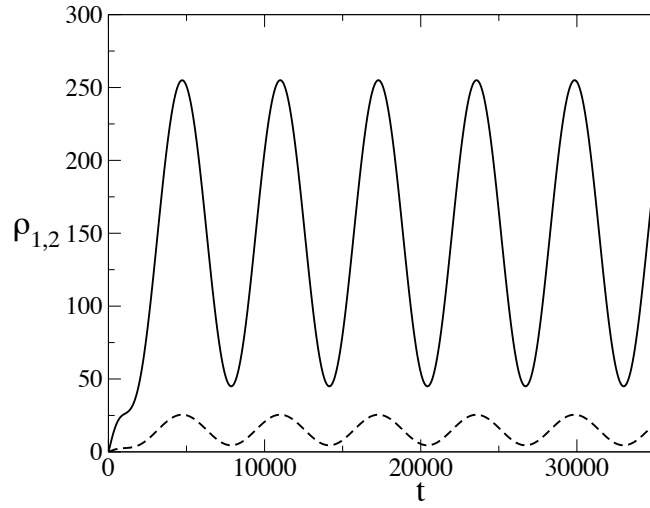


Figure 3: Illustration of a periodic motion of the non-trivial special point, i.e., motion of the components $\rho_0^{(t)}, \rho_1^{(t)}$ of the special point from Eq.(8). Solid line: $\rho_0^{(t)}$; dashed line: $\rho_1^{(t)}$. Initial values: $\rho_0^{(1)} = 6.0 \cdot 10^{-2}$; $\rho_1^{(2)} = 2.0 \cdot 10^{-1}$; the parameters $f_{10} = 10^{-4}$; $f_{01} = 0$; $r_1 = -10^{-3}$; $b_{01} = -0.1$; $b_{10} = 0$ are assumed to be constant, but $r_0 = 1.5 \tanh(10^{-3}t) - 1.05 \sin(10^{-3}t)$. N.B. the y -axis should read $\rho_{0,1}$ and not $\rho_{1,2}$.

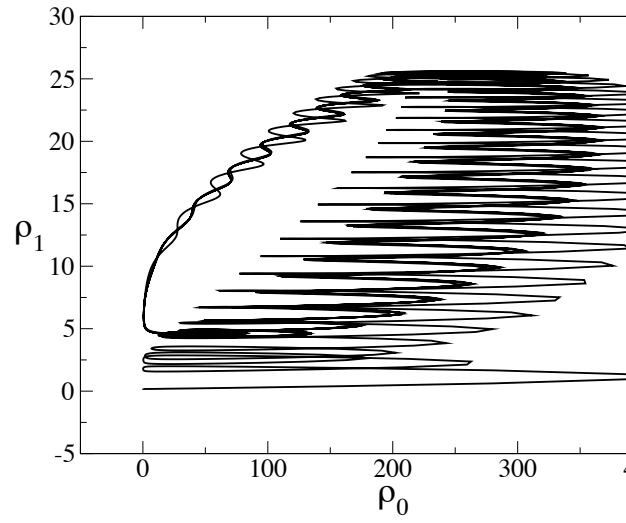


Figure 4: Illustration of the corresponding complicated trajectory of the system in the phase space from Eq. (8). All values of the parameters and quantities are chosen in order to make the illustration more visible. Initial values: $\rho_0^{(0)} = 6.0 \cdot 10^{-2}$; $\rho_1^{(0)} = 2.0 \cdot 10^{-1}$; the parameters $f_{10} = 10^{-4}$; $f_{01} = 0$; $r_1 = -10^{-3}$; $b_{01} = -0.1$; $b_{10} = 0$ are assumed to be constant, but $r_0 = 1.5 \tanh(10^{-3}t) - 1.05 \sin(10^{-3}t)$

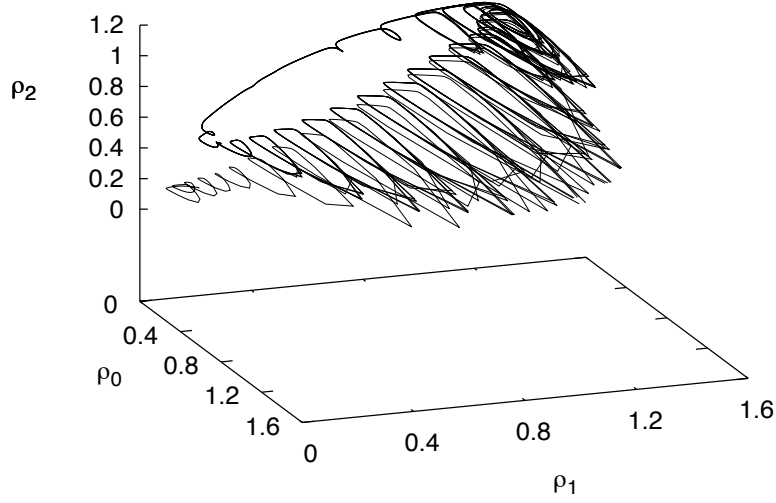


Figure 5: Illustration of the trajectory of a system made of three populations, i.e. non-believers and followers of two distinct ideologies. The initial values for the run are: $\rho_0 = 1$; $\rho_1 = 0.2$; $\rho_2 = 0.1$. Other parameters have the constant values: $f_{10} = 10^{-2}$; $f_{12} = 10^{-5}$; $f_{01} = 0$; $f_{02} = 10^{-6}$; $f_{20} = 10^{-3}$; $f_{21} = 10^{-3}$; ; $r_1 = -10^{-3}$; $r_2 = -10^{-3}$; $b_{01} = -0.1$; $b_{10} = 0$; $b_{02} = -10^{-4}$; $b_{12} = 10^{-6}$; $b_{20} = 10^{-5}$; $b_{21} = -10^{-4}$, but $r_0 = 1.5 \tanh(0.001t) - 1.05 \sin(0.001t) + 0.02 \sin(0.03t)$. The trajectory is calculated up to $t = 40000$